Collective states of the odd-mass nuclei within the framework of the Interacting Vector Boson Model

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Abstract.

A supersymmetric extension of the dynamical symmetry group $Sp^B(12,R)$ of the Interacting Vector Boson Model (IVBM), to the orthosymplectic group $OSp(2\Omega/12,R)$ is developed in order to incorporate fermion degrees of freedom into the nuclear dynamics and to encompass the treatment of odd mass nuclei. The bosonic sector of the supergroup is used to describe the complex collective spectra of the neighboring even-even nuclei and is considered as a core structure of the odd nucleus. The fermionic sector is represented by the fermion spin group $SO^F(2\Omega) \supset SU^F(2)$.

The so obtained, new exactly solvable limiting case is applied for the description of the nuclear collective spectra of odd mass nuclei. The theoretical predictions for different collective bands in three odd mass nuclei, namely ^{157}Gd, ^{173}Yb and ^{163}Dy from rare earth region are compared with the experiment. The $B(E2)$ transition probabilities for the ^{157}Gd and ^{163}Dy between the states of the ground band are also studied. The important role of the symplectic structure of the model for the proper reproduction of the $B(E2)$ behavior is revealed. The obtained results reveal the applicability of the models extension.

PACS 21.60.Fw, 21.60.Ev, 21.10.Re, 27.70.+q, 23.20.-g

1. Introduction

Symmetry is an important concept in nuclear physics. In finite many-body systems of this type, it appears as time reversal, parity, and rotational invariance, but also in the form of dynamical symmetries.

Many collective properties of the nuclei have been investigated using models based on dynamical groups. One of the most popular and widely used models of this type are the Interacting Boson Model (IBM) [1] and its extensions [2],[3] as well as the symplectic model [4] based on the group $Sp(6, R)$. In these, algebraic models the bands of collective states are classified by the irreducible representations (irreps) of the corresponding chains of groups and their corresponding properties, such as energy levels and electromagnetic transition strengths, are determined by algebraic methods.

It is well known that nucleons have intrinsic spin and that there are strong spin-orbit interactions. Moreover, the experiment reveals, that the presence of spin does not prevent the appearance of rotational bands. It also establishes similarity in the rotational character of the different collective bands for neighboring even-even and odd-even nuclei far from closed shells. For the description of the nuclear spectra of such even-even nuclei the above mentioned variety of boson models is used. This is possible because in the even-even nuclei the pairs of nucleons are usually considered as coupled to integer angular momentum. However, this is not the case for odd mass nuclei. Thus, the following question naturally arises: how to incorporate fermion degrees of freedom into the nuclear dynamics in a way that the rotational character of the collective bands is preserved.

In general, it is believed that the collective states of odd nuclei can be described by using particle-core coupled-type models. The natural extension of IBM, the Interacting Boson-Fermion Model (IBFM) [5], which includes single-particle (fermion) degrees of freedom in addition to the collective (boson) ones, have provided in the last decays a unified framework for the description of even-even and odd-even nuclei distant from closed shell configurations, at least in the low-angular momentum domain.

For the description of odd$-A$ nuclei, a fermion needs to be coupled to the N boson system. This can be done by a semimicroscopical approach which relies on seniority in the nuclear shell model [2]. As an alternative to this, in the IBFM approach, Hamiltonians exhibiting dynamical Bose-Fermi symmetries, that are analytically solvable [5] are constructed. Thus, the extension of the IBM for the case of odd mass nuclei leads to the group structure $U^B(6) \otimes U^F(m)$ (IBFM-1) or $U^B_\pi(6) \otimes U^B_\nu(6) \otimes U^F(m)$ (IBFM-2), where $m = \sum_j(2j + 1)$ is the dimension of the single-particle space. Obviously, in the general case for arbitrary $m-$ values, analytical expressions for the nuclear levels would be too cumbersome and will contain too many parameters. Moreover, orbitals higher in energy than those of the valence shell might play a role and have to be considered (for example, in the $Sp(6, R)$ model), thus breaking the symmetric scheme. Therefore, numerical calculations have to be performed with schematic Hamiltonians. These deficiencies, motivate the development of the new

extension of the IVBM, which will be based on the success of the boson description of the even-even nuclei, but will include the fermion degrees of freedom in a simple and straightforward way, that still leads to exact analytic solutions.

In the early 1980s, a boson-number-preserving version of the phenomenological algebraic Interacting Vector Boson Model (IVBM) [6] was introduced and applied successfully [7] to a description of the low-lying collective rotational spectra of the even-even medium and heavy mass nuclei. With the aim of extending these applications to incorporate new experimental data on states with higher spins and to incorporate new excited bands, we explored the symplectic extension of the IVBM [8], for which the dynamical symmetry group is $Sp(12, R)$. This extension is realized from, and has its physical interpretation over basis states of its maximal compact subgroup $U(6) \subset Sp(12, R)$, and resulted in the description of various excited bands of both positive and negative parity of complex systems exhibiting rotation-vibrational spectra [9]. With the present work we extend the earlier applications of IVBM for the description of the ground and first excited positive and/or negative bands of odd mass nuclei. In order to do this we propose a new dynamical symmetry which is applied to real odd nuclear systems.

Thus, it is the purpose of this paper to bring spin explicitly into the symplectic IBVM. We approach the problem by considering the simplest physical picture in which a particle (or quasiparticle) with intrinsic spin taking a single $j-$value j is coupled to an even-even nucleus whose states belong to an $Sp(12, R)$ irrep. Nevertheless, the results for the energy spectra and the intraband transitions between the states of the ground state band obtained in this simplified version of the model agree rather well with the experimental data.

2. The even-even core

The IVBM is based on the introduction of two kinds of vector bosons (called p and n bosons), that "built up" the collective excitations in the nuclear system. The creation operators of these bosons are assumed to be $SO(3)$ vectors and they transform according to two independent fundamental representations (1,0) of the group $SU(3)$. These bosons form a "pseudospin" doublet of the $U(2)$ group and differ in their "pseudospin" projection $\alpha = \pm\frac{1}{2}$. We want to point out that these vector bosons should be considered as "building blocks" generating appropriate algebraic structures rather than real correlated fermion pairs coupled to angular momentum $l = 1$.

The algebraic structure of the IVBM is realized in terms of creation and annihilation operators $u_m^+(\alpha)$, $u_m(\alpha)$ $(m = 0, \pm 1)$. The later are related to the cyclic coordinates $x_{\pm 1}(\alpha) = \mp\frac{1}{\sqrt{2}}(x_1(\alpha) \pm ix_2(\alpha)), x_0(\alpha) = x_3(\alpha)$, and their associated momenta $q_m(\alpha) = -i\partial/\partial x^m(\alpha)$ in the standard way

$$u_m^+(\alpha) = \frac{1}{\sqrt{2}}(x_m(\alpha) - iq_m(\alpha)), \qquad (1)$$
$$u_m(\alpha) = (u_m^+(\alpha))^\dagger),$$

where $x_i(\alpha)$ $i = 1, 2, 3$ are Cartesian coordinates of a quasi-particle vectors with an additional index - the projection of the "pseudo-spin" $\alpha = \pm\frac{1}{2}$. The bilinear products of the creation and annihilation operators of the two vector bosons (1) generate the boson representations of the non-compact symplectic group $Sp(12, R)$ [6]:

$$F^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u^+_k(\alpha) u^+_m(\beta),$$

$$G^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u_k(\alpha) u_m(\beta), \tag{2}$$

$$A^L_M(\alpha, \beta) = \sum_{k,m} C^{LM}_{1k1m} u^+_k(\alpha) u_m(\beta), \tag{3}$$

where C^{LM}_{1k1m}, which are the usual Clebsch-Gordon coefficients for $L = 0, 1, 2$ and $M = -L, -L + 1, ...L$, define the transformation properties of (2) and (3) under rotations. The commutation relations between the pair creation and annihilation operators (2) and the number preserving operators (3) are given in [6].

Being a noncompact group, the representations of $Sp(12, R)$ are of infinite dimension, which makes it impossible to diagonalize the most general Hamiltonian. When restricted to the group $U^B(6)$, each irrep of the group $Sp^B(12, R)$ decomposes into irreps of the subgroup characterized by the partitions [8],[10]:

$$[N, 0^5]_6 \equiv [N]_6,$$

where $N = 0, 2, 4, \ldots$ (even irrep) or $N = 1, 3, 5, \ldots$ (odd irrep). The subspaces $[N]_6$ are finite dimensional, which simplifies the problem of diagonalization. Therefore the complete spectrum of the system can be calculated through the diagonalization of the Hamiltonian in the subspaces of all the unitary irreducible representations (UIR) of $U(6)$, belonging to a given UIR of $Sp(12, R)$, which further clarifies its role of a group of dynamical symmetry. Since N is the number of collective excitations (phonons) rather than real nucleon pairs, in the present paper we consider only the even irrep of $Sp(12, R)$.

The most general one and two-body Hamiltonian can be expressed in terms of symplectic generators. In general, such rather general Hamiltonian has to be diagonalized numerically to obtain the energy eigenvalues and wave functions. There exist, however, special situations in which the eigenvalues can be obtained in closed, analytical form. These special solutions provide a framework in which energy spectra and other nuclear properties can be interpreted in a qualitative way. These situations correspond to dynamical symmetries of the Hamiltonian.

The Hamiltonian, corresponding to the unitary limit of IVBM [8]

$$Sp(12, R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes (U(1) \otimes U(1)), \tag{4}$$

expressed in terms of the first and second order invariant operators of the different subgroups in the chain (4) is [8]:

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + \alpha_1 T^2_0. \tag{5}$$

H (5) is obviously diagonal in the basis

$$\mid [N]_6; (\lambda, \mu); KLM; T_0\rangle \equiv \mid (N, T); KLM; T_0\rangle, \tag{6}$$

labelled by the quantum numbers of the subgroups of the chain (4). Its eigenvalues are the energies of the basis states of the boson representations of $Sp(12,R)$:

$$E((N,T),L,T_0) = aN + bN^2 + \alpha_3 T(T+1) \\ + \beta_3 L(L+1) + \alpha_1 T_0^2. \tag{7}$$

The non-compact group $Sp(12,R)$ has a Jordan (three grading) decomposition with respect to its maximal compact subgroup $U(6)$. Its Lie algebra g can be decomposed as a vector space direct sum:

$$g = g_- \oplus g_0 \oplus g_+.$$

Every unitary lowest weight representation of $Sp(12,R)$ can be constructed by acting consequently on the boson lowest weight state (LWS) $\mid \Omega \rangle_B$, transforming in a definite $U(6)$ representation, with the raising generators $F_M^L(\alpha,\beta)$ which belong to the g_+ space. This action generates an infinite set of states (6) that form the basis of a unitary lowest weight representation of $Sp(12,R)$. If the LWS $\mid \Omega \rangle_B$ transforms irreducibly under $U(6)$, then the corresponding unitary representation of $Sp(12,R)$ is also irreducible. The unitary lowest weight irreducible representation of $Sp(12,R)$ can therefore be uniquely labeled by the $U(6)$ labels of their lowest weight states. In the boson space there are only two nonequivalent irreducible lowest weight states, namely, the (boson) vacuum

$$\mid \Omega \rangle_B = \mid 0 \rangle_B \tag{8}$$

and the "one-particle" state

$$\mid \Omega \rangle_B = u_k^{\dagger}(\alpha) \mid 0 \rangle_B \ . \tag{9}$$

The construction of the symplectic basis for the even IR of $Sp(12,R)$, which can be obtained by action of the fully symmetric coupled powers of raising operators $F_M^L(\alpha,\beta)$ on the on vacuum state (8), is given in details in [8]. The $Sp(12,R)$ classification scheme for the $SU(3)$ boson representations for even value of the number of bosons N is shown on Table I in Ref. [8] (see also Table 1).

The most important application of the $U^B(6) \subset Sp^B(12,R)$ limit of the theory is the possibility it affords for describing both even and odd parity bands up to very high angular momentum [8]. In order to do this we first have to identify the experimentally observed bands with the sequences of basis states of the even $Sp(12,R)$ irrep (Table 1). As we deal with the symplectic extension of the boson representations of the number preserving $U^B(6)$ symmetry we are able to consider all even eigenvalues of the number of vector bosons N with the corresponding set of pseudospins T, which uniquely define the $SU^B(3)$ irreps (λ,μ). The multiplicity index K appearing in the final reduction to the $SO(3)$ is related to the projection of L on the body fixed frame and is used with the parity (π) to label the different bands (K^{π}) in the energy spectra of the nuclei. For the even-even nuclei we have defined the parity of the states as $\pi_{core} = (-1)^T$ [8]. This allowed us to describe both positive and negative bands.

Further, we use the algebraic concept of "yrast" states, introduced in [8]. According to this concept we consider as yrast states the states with given L, which minimize the

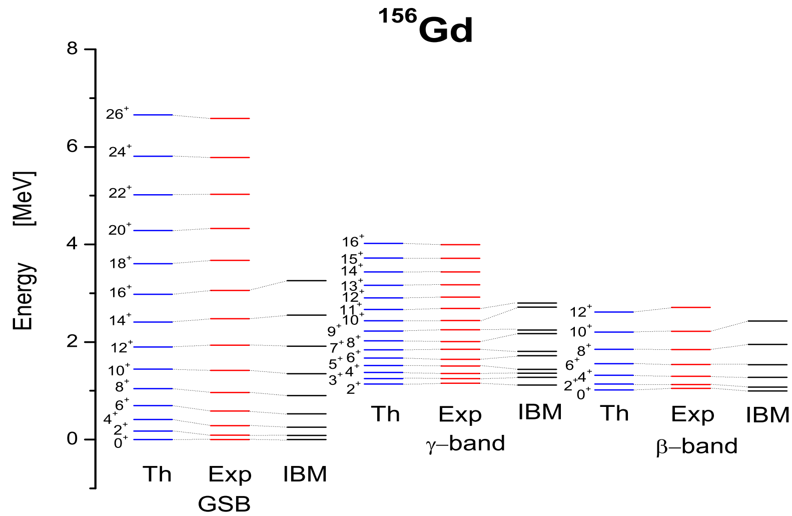

Figure 1. Comparison of the theoretical and experimental energies for the ground and first excited bands of ^{156}Gd.

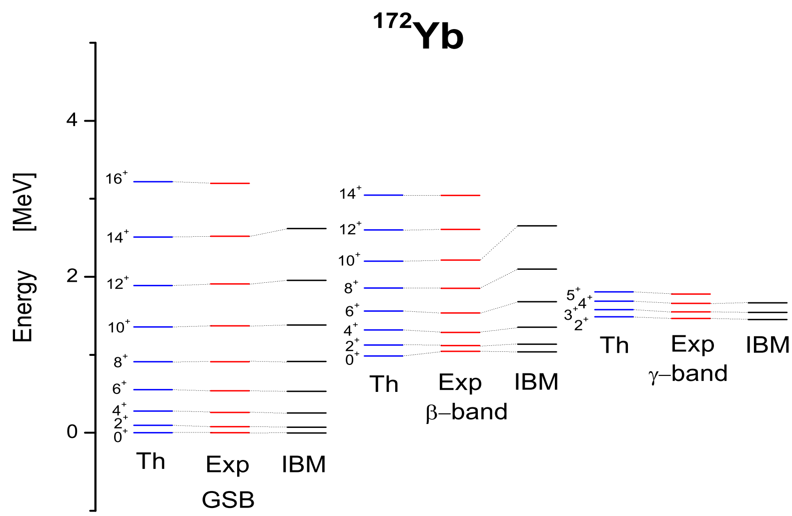

Figure 2. The same as Fig. 1, but for ^{172}Yb.

energy (7) with respect to the number of vector bosons N that build them. Thus the states of the ground state band (GSB) were identified with the $SU(3)$ multiplets $(0, \mu)$ [8]. In terms of (N, T) this choice corresponds to $(N = 2\mu, T = 0)$ and the sequence of states with different numbers of bosons $N = 0, 4, 8, \ldots$ and pseudospin $T = 0$ (and also $T_0 = 0$). Hence the minimum values of the energies (7) are obtained at $N = 2L$.

The presented mapping of the experimental states onto the $SU(3)$ basis states, using the algebraic notion of yrast states, is a particular case of the so called "stretched"

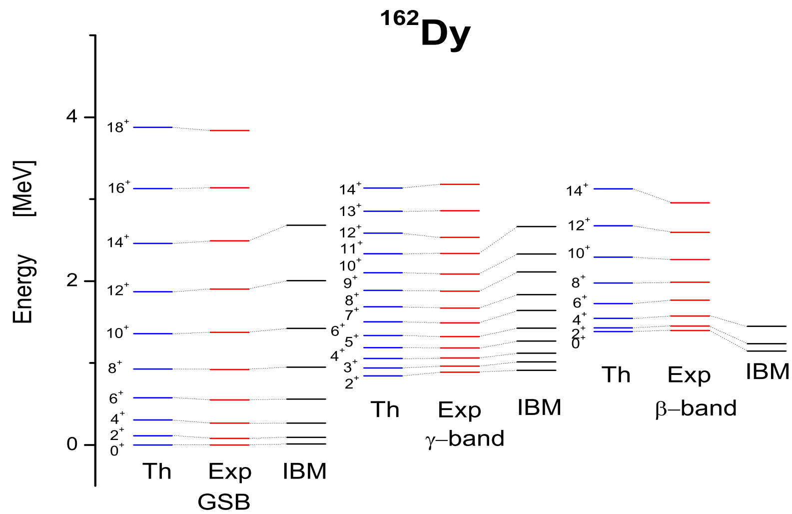

Figure 3. The same as Fig. 1, but for ^{162}Dy.

states [11]. The latter are defined as the states with $(\lambda_0 + 2k, \mu_0)$ or $(\lambda_0, \mu_0 + k)$, where $N_i = \lambda_0 + 2\mu_0$ and $k = 0, 1, 2, 3, \ldots$.

It was established [12] that the correct placement of the bands in the spectrum strongly depends on their bandheads configuration, and in particular, on the minimal or initial number of bosons, $N = N_i$, from which they are built. The latter determines the starting position of each excited band.

Thus, for the description of the different excited bands, we first determine the N_i of the band head structure and develop the corresponding excited band over the stretched $SU(3)$ multiplets. This corresponds to the sequence of basis states with $N = N_i, N_i{+}4, N_i{+}8, \ldots (\Delta N = 4)$. The values of T for the first type of stretched states ($\lambda-$changed) are changed by step $\Delta T = 2$, whereas for the second type ($\mu-$changed) $-T$ is fixed so that in both cases the parity is preserved even or odd, respectively. For all presented even-even nuclei, the states of the corresponding $\beta-$ and $\gamma-$ bands are associated with the stretched states of the first type ($\lambda-$ changed).

The odd-A nuclei ^{157}Gd, ^{173}Yb and ^{163}Dy, to which we apply our model, can be considered as a particle coupled to the even-even cores ^{156}Gd, ^{172}Yb and ^{162}Dy, respectively. We determine the values of the five phenomenological model parameters $a, b, \alpha_3, \beta_3, \alpha_1$ by fitting the energies of the ground and few excited bands ($\gamma-$ and/or $\beta-$ bands) of the even-even nuclei to the experimental data [13], using a χ^2 procedure. The theoretical predictions for the even core nuclei are presented in the Figures 1–3. For comparison, the predictions of IBM (with 4 adjustable parameters) are also shown. The IBM results for ^{156}Gd and ^{162}Dy,^{172}Yb are extracted from Refs. [14] and [15], respectively. From the figures one can see that the calculated energy levels agree rather well up to very high angular momenta with the observed data. One can see also that

for high spins ($L \geq 10-14$), where the deviations of the IBM predictions become more significant, the structure of the energy levels of the GSB ($\beta-$ and $\gamma-$bands) is reproduced rather well.

3. The inclusion of spin

Underlying the conventional nuclear shell model is the idea that the low-lying states of nuclei can be restricted to a valence space of states obtained by putting nucleons into a finite set of single-particle states indexed $i = 1, \ldots, \Omega$; i. e. the M valence-particle Hilbert space is the anti-symmetrized (exterior) product of M copies of an $\Omega-$dimensional single-nucleon Hilbert space. This space carries a sum of two irreducible representations of the fermion pair algebra $SO(2\Omega)$ [16]. The set of all even fermion states span an irreducible representation of the $SO(2\Omega)$ algebra and the set of all states of odd fermion number span another irreducible representation.

Thus, in order to incorporate the intrinsic spin degrees of freedom into the symplectic IVBM, we extend the dynamical algebra of $Sp(12, R)$ to the orthosymplectic algebra of $OSp(2\Omega/12, R)$. For this purpose we introduce a particle (quasiparticle) with spin j and consider a simple core plus particle picture. Thus, in addition to the boson collective degrees of freedom (described by dynamical symmetry group $Sp(12, R)$) we introduce creation and annihilation operators $a^{\dagger}_{m}$ and a_m ($m = -j, \ldots, j$), which satisfy the anticommutation relations

$$\begin{aligned} &\{a^{\dagger}_{m}, a^{\dagger}_{m'}\} = \{a_m, a_{m'}\} = 0, \\ &\{a_m, a^{\dagger}_{m'}\} = \delta_{mm'}. \end{aligned} \tag{10}$$

All bilinear combinations of a^{+}_{m} and $a_{m'}$, namely

$$\begin{aligned} f_{mm'} &= a^{\dagger}_{m} a^{\dagger}_{m'}, \quad m \neq m' \\ g_{mm'} &= a_m a_{m'}, \quad m \neq m'; \end{aligned} \tag{11}$$

$$C_{mm'} = (a^{\dagger}_{m} a_{m'} - a_{m'} a^{\dagger}_{m})/2 \tag{12}$$

generate the (Lie) fermion pair algebra of $SO^F(2\Omega)$. Their commutation relations are:

$$\begin{aligned} [g_{mn}, C_{m'n'}] &= \delta_{nm'} g_{mn'} - \delta_{mm'} g_{nn'}, \\ [C_{mn}, f_{n'n'}] &= \delta_{nm'} g_{mn'} - \delta_{nm'} g_{mn'}, \\ [g_{mn}, f_{m'n'}] &= -\delta_{mm'} C_{n'n} - \delta_{nn'} C_{n'm} + \delta_{n'm} C_{n'n} + \delta_{m'n} C_{n'm}, \end{aligned}$$

The number preserving operators (12) generate maximal compact subalgebra of $SO^F(2\Omega)$, i.e. $U^F(\Omega)$. The upper (lower) script B or F denotes the boson or fermion degrees of freedom, respectively.

Making use of the embedding $SU^F(2) \subset SO^F(2\Omega)$, we make orthosymplectic

(supersymmetric) extension of the IVBM which is defined through the chain:

$$
\begin{array}{ccccc}
OSp(2\Omega/12,R) & \supset & SO^F(2\Omega) & \otimes & Sp^B(12,R) \\
 & & & & \Downarrow \\
 & & \Downarrow & \otimes & U^B(6) \\
 & & & & N \\
 & & & & \Downarrow \\
 & & SU^F(2) & \otimes & SU^B(3)\otimes U^B_T(2) \\
 & & j & & (\lambda,\mu) \Longleftrightarrow (N,T) \\
 & & & \searrow & \Downarrow \\
 & & & \otimes & SO^B(3)\otimes U(1) \\
 & & & & L \qquad\qquad T_0 \\
 & & & \Downarrow & \\
 & & Spin^{BF}(3) & \supset & Spin^{BF}(2), \\
 & & J & & J_0
\end{array}
\qquad (13)
$$

where bellow the different subgroups the quantum numbers characterizing their irreducible representations are given. Here with $Spin^{BF}(n)$ $(n = 2,3)$ is denoted the universal covering group of the $SO(n)$. From (13) it can be seen that the coupling of the boson and fermion degrees of freedom is done on the level of the angular momenta. We want to stress, however, that although the formal "coupling" is done at the "final" stage, the present situation is not identical to that of IBFM. In the latter the even-even core, to which an odd unpaired nucleon is coupled to, is considered as "inert". In the present approach since the (ortho)symplectic structures are taken into account (allowing for the change of number of phonon excitations N), the core is not anymore inert. Physically, this does not correspond to the weak coupling limit (as should be if N was fixed) between the core and particle as it is in the case of IBFM (on this level of coupling).

4. Application of the new dynamical symmetry

4.1. The energy spectrum

In this paper we expand the earlier application of the IVBM [8], developed for the description of the collective bands of even-even nuclei, in order to include in our considerations the case of odd mass nuclei.

We can label the basis states according to the chain (13) as:

$$
\mid [N]_6; (\lambda,\mu); KL; j; JJ_0; T_0 \,\rangle \equiv \mid [N]_6; (N,T); KL; j; JJ_0; T_0 \,\rangle, \qquad (14)
$$

where $[N]_6-$ is the $U(6)$ labeling quantum number, $(\lambda,\mu)-$ are the $SU(3)$ quantum numbers characterizing the core excitations, K is the multiplicity index in the reduction $SU(3) \subset SO(3)$, L is the core angular momentum, $j-$the spin of the odd particle, J, J_0 are the total (coupled) angular momentum and its third projection, and T,T_0 are the pseudospin and its third projection, respectively. Since the $SO(2\Omega)$ label is irrelevant for our application, we drop it in the states (14).

The Hamiltonian can be written as linear combination of the Casimir operators of the different subgroups in (13):

$$H = aN + bN^2 + \alpha_3 T^2 + \beta_3' L^2 + \alpha_1 T_0^2 + \eta j^2 + \gamma' J^2 + \zeta J_0^2 \tag{15}$$

and it is obviously diagonal in the basis (14) labeled by the quantum numbers of their representations. Then the eigenvalues of the Hamiltonian (15), that yield the spectrum of the odd mass system are:

$$\begin{aligned} E(N;T,T_0;L,j;J,J_0) = {} & aN + bN^2 + \alpha_3 T(T+1) + \beta_3' L(L+1) + \alpha_1 T_0^2 \\ & + \eta j(j+1) + \gamma' J(J+1) + \zeta J_0^2. \end{aligned} \tag{16}$$

We note that only the last three terms of (15) come from the orthosymplectic extension. But since only one fermion ($M = 1$) is considered (and j is fixed), the $j-$term in (16) is just additive constant and can be dropped. (The presence of the latter should only rescale the values of the rest model parameters.) Thus, for the description of the excitation spectra of odd-mass nuclei only two new parameters are involved in the fitting procedure. We choose parameters $\beta_3' = \frac{1}{2}\beta_3$ and $\gamma' = \frac{1}{2}\gamma$ instead of β_3 and γ in order to obtain the Hamiltonian form of ref. [8] (setting $\beta_3 = \gamma$), when for the case $j = 0$ (hence $J = L$) we recover the symplectic structure of the IVBM.

The infinite set of basis states classified according to the reduction chain (13) are schematically shown in Table 1. The fourth and fifth columns show the $SO^B(3)$ content of the $SU^B(3)$ group, given by the standard Elliott's reduction rules [17], while in the next column are given the possible values of the common angular momentum J, obtained by coupling of the orbital momentum L with the spin j. The latter is vector coupling and hence all possible values of the total angular momentum J should be considered. For simplicity, only the maximally aligned ($J = L + j$) and maximally antialigned ($J = L - j$) states are illustrated in Table 1.

The basis states (14) can be considered as a result of the coupling of the orbital $\mid (N,T); KLM; T_0\rangle$ (6) and spin ϕ_{jm} wave functions. Then, if the parity of the single particle is π_{sp}, the parity of the collective states of the odd$-A$ nuclei will be $\pi = \pi_{core}\pi_{sp}$. Thus, the description of the positive and/or negative parity bands requires only the proper choice of the core band heads, on which the corresponding single particle is coupled to, generating in this way the different odd$-A$ collective bands. Our choice is based on the fact, which has been always understood in nuclear physics, that well defined rotational bands can exist only when they are adiabatic relative to other degrees of freedom. In this way (in adiabatic approximation) the single particle is dragged around in the core field (which corresponds to the "strong" coupling limit as is in our case) and the combined system is essentially a new rotor with slightly different bulk properties, such as moment of inertia, etc.

Further in the present considerations, the yrast conditions yield relations between the number of bosons N and the coupled angular momentum J that characterizes each

Table 1. Classification scheme of basis states (14) according the decompositions given by the chain (13).

N	T	(λ,μ)	K	L	$J=L\pm j$
0	0	$(0,0)$	0	0	j
2	1	$(2,0)$	0	$0,2$	$j;\ 2\pm j$
	0	$(0,1)$	0	1	$1\pm j$
	2	$(4,0)$	0	$0,2,4$	$j;\ 2\pm j;4\pm j$
4	1	$(2,1)$	1	$1,2,3$	$1\pm j;\ 2\pm j;\ 3\pm j$
	0	$(0,2)$	0	$0,2$	$j;\ 2\pm j$
	3	$(6,0)$	0	$0,2,4,6$	$j;\ 2\pm j;\ 4\pm j;6\pm j$
	2	$(4,1)$	1	$1,2,3,4,5$	$1\pm j;\ 2\pm j;\ 3\pm j;$ $4\pm j;\ 5\pm j$
6	1	$(2,2)$	2	$2,3,4$	$2\pm j;\ 3\pm j;\ 4\pm j$
			0	$0,2$	$j;\ 2\pm j$
	0	$(0,3)$	0	$1,3$	$1\pm j;\ 3\pm j$
	4	$(8,0)$	0	$0,2,4,6,8$	$j;\ 2\pm j;\ 4\pm j;$ $6\pm j;\ 8\pm j$
	3	$(6,1)$	1	$1,2,3,4,5,6,7$	$1\pm j;\ 2\pm j;\ 3\pm j;$ $4\pm j;\ 5\pm j;6\pm j;$ $7\pm j;8\pm j$
	2	$(4,2)$	2	$2,3,4,5,6$	$2\pm j;\ 3\pm j;\ 4\pm j;$ $5\pm j;\ 6\pm j$
8			0	$0,2,4$	$j;\ 2\pm j;\ 4\pm j$
	1	$(2,3)$	2	$2,3,4,5$	$2\pm j;\ 3\pm j;\ 4\pm j;5\pm j$
			0	$1,3$	$1\pm j;\ 3\pm j$
	0	$(0,4)$	0	$0,2,4$	$j;2\pm j;\ 4\pm j$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

collective state. For example, the collective states of the GSB $K^{\pi}_{J}=\frac{3}{2}^{-}$ are identified with the $SU(3)$ multiplets $(0,\mu)$ which yield the sequence $N=2(J-j)=0,2,4,\ldots$ for the corresponding values $J=\frac{3}{2},\frac{5}{2},\frac{7}{2},\ldots$. The pseudospin for the $SU(3)$ multiplets $(0,\mu)$ is $T=0$ and hence $\pi_{core}=(-1)^{T}=(+)$. Here it is assumed that the single particle has $j=3/2$ and parity $\pi_{sp}=(-)$, so that the common parity π is also negative.

For the description of the different excited bands, we first determine the N_i of the band head structure and then we map the states of the corresponding band onto the sequence of basis states with $N=N_i, N_i+2, N_i+4, \ldots$ $(\Delta N=2)$ and $T=even=fixed$ or $T=odd=fixed$, respectively. This choice corresponds to the stretched states of the second type ($\mu-$changed).

We will point out that the (ortho)symplectic structure of the model space gives us rather rich possibilities to map experimentally observed states onto the basis states.

Thus, another possibility of developing the sequence of band's states is to take again $N = N_i, N_i+4, N_i+8, \ldots$ ($\Delta N = 4$) but to change $T = T_i, T_i+2, T_i+4, \ldots$ ($\Delta T = 2$) in such a way, that the parity is preserved even or odd, respectively. Such correspondence takes place for the first type of the stretched states ($\lambda-$changed). In the present application, all the collective bands under consideration are associated with the stretched states of second type ($\mu-$changed).

The number of adjustable parameters needed for the complete description of the collective spectra of the odd-A nuclei is two, namely γ and ζ. They are evaluated by a fit to the experimental data [13] of the GSB of the corresponding odd-A nucleus. The comparison between the experimental spectra for the GSB and first few excited bands and our calculations using the values of the model parameters given in Table 2 for the nuclei ^{157}Gd, ^{173}Yb and ^{163}Dy is illustrated in Figures 4$-$6. The last single particle, which for all of these rare earth nuclei is a neutron, occupies the major shell $N = 82-126$, where the relevant single particle levels are $2f_{\frac{7}{2}}, 2f_{\frac{5}{2}}, 3p_{\frac{3}{2}}, 3p_{\frac{1}{2}}$ having odd parity ($\pi_{sp} = -$) (excluding the intruder from the upper shell with opposite parity). In our considerations we take into account only the first available single particle orbit j (generating the group $SO(2\Omega)$ with $\Omega = (2j+1)$), which for the first nucleus implies $j = \frac{3}{2}$, while for the other two $-$ $j = \frac{5}{2}$. The Nilsson asymptotic quantum numbers $\Omega[Nn_3\Lambda]$ are written bellow each band. One can see from the figures that the calculated energy levels agree rather well in general with the experimental data up to very high angular momenta. For comparison, in the Figures 4$-$6 the IBFM results (obtained by total 7 adjustable parameters) are also shown. They are extracted from Refs. [14] and [15], respectively. Note that all calculated levels, for the bands considered, are in correct order in contrast to IBFM results (for ^{157}Gd). Another difference between the IVBM and IBFM predictions is that in the former the correct placement of all the band heads is reproduced quite well.

In the Table 2, the values of N_i, T, T_0, J, J_0 and χ^2 for each band under consideration are also given.

4.2. Electromagnetic transition probabilities

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. Calculation of the latter is a good test of the nuclear model functions. The most important electromagnetic features are the $E2$ transitions. In this subsection we discuss the calculation of the $E2$ transition strengths and compare the results with the available experimental data.

As was mentioned, in the symplectic extension of the IVBM the complete spectrum of the system is obtained in all the even subspaces with fixed N- even of the UIR $[N]_6$ of $U(6)$, belonging to a given even UIR of $Sp(12,R)$. The classification scheme of the $SU(3)$ boson representations for even values of the number of bosons N was presented in Table 1.

In this paper we give as an example the evaluation of the $E2$ transition probabilities

Table 2. Values of the model parameters.

Nucl.	*bands*	N_i	T	T_0	J	J_0	χ^2	*parameters*
^{157}Gd	$GSB: K^\pi = 3/2^-$	0	0	0	$L+j$	$3/2$	0.00100	$a = 0.03225$ $b = -0.00075$ $\alpha_3 = 0.00332$
$j = 3/2$	$K^\pi = 5/2^-$	20	8	4	$L-j$	$5/2$	0.00014	$\beta_3 = 0.00998$ $\alpha_1 = -0.00303$
	$K^\pi = 1/2^-[530]$	20	10	3	$L-j$	$1/2$	0.00061	$\gamma = 0.00806$
	$K^\pi = 1/2^-[521]$	24	12	6	$L-j$	$1/2$	0.00018	$\zeta = -0.03549$
^{173}Yb	$GSB: K^\pi = 5/2^-$	0	0	0	$L+j$	$5/2$	0.00018	$a = 0.00716$ $b = -0.00027$ $\alpha_3 = 0.00153$
$j = 5/2$	$K^\pi = 7/2^-$	40	20	5	$L-j$	$7/2$	0.000002	$\beta_3 = 0.01198$ $\alpha_1 = -0.00605$
	$K^\pi = 3/2^-$	72	36	3	$L-j$	$3/2$	0.00034	$\gamma = 0.01281$
	$K^\pi = 5/2^-[523]$	60	30	0	$L-j$	J	0.000007	$\zeta = -0.00555$
^{163}Dy	$GSB: K^\pi = 5/2^-$	0	0	0	$L+j$	$5/2$	0.000004	$a = 0.01242$ $b = 0.00041$ $\alpha_3 = 0.00170$
$j = 5/2$	$K^\pi = 5/2^+$	22	11	3	$L+j$	$5/2$	0.00008	$\beta_3 = 0.01159$ $\alpha_1 = -0.00658$
	$K^\pi = 1/2^-$	32	16	5	$L-j$	$1/2$	0.00007	$\gamma = 0.01124$ $\zeta = -0.00779$

between the states of the ground state bands $K_J^\pi = \frac{3}{2}^-$ and $K_J^\pi = \frac{5}{2}^-$. For both cases, the states of the GSB are identified with the $SU(3)$ multiplets $(0,\mu)$ and $\mu = L$. This yields the sequence $N = 2(J-j) = 0, 2, 4, \ldots$ for the corresponding values $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$. In terms of (N,T) this corresponds to $(N = 2\mu, T = 0)$.

Using the tensorial properties of the $Sp(12,R)$ generators with respect to (4) it is easy to define the $E2$ transition operator [18] between the states of the considered band as:

$$T^{E2} = e\left[A^{[1-1]_6}_{[210]_3[0]_2}{}^{20}_{00} + \theta([F\times F]^{[4]_6}_{(0,2)[0]_2}{}^{20}_{00} + [G\times G]^{[-4]_6}_{(2,0)[0]_2}{}^{20}_{00}\right]. \quad (17)$$

The first part of (17) is a $SU(3)$ generator and actually changes only the angular momentum with $\Delta L = 2$.

The tensor product

$$[F\times F]^{[4]_6}_{(0,2)[0]_2}{}^{20}_{00} = \sum C^{[2]_6\ \ [2]_6\ \ \ [4]_6}_{(2,0)[2]_2\,(2,0)[2]_2\ (0,2)[0]_2} C^{(2,0)\ (2,0)\ \ (0,2)}_{(2)_3\ \ (2)_3\ \ (2)_3} \times C^{20}_{20\ 20} C^{10}_{11\ 1-1}\, F^{[2]_6}_{(2,0)[2]_2}{}^{20}_{11} F^{[2]_6}_{(2,0)[2]_2}{}^{20}_{1-1} \quad (18)$$

of the operators (2) that are the pair raising $Sp(12,R)$ generators changes the number

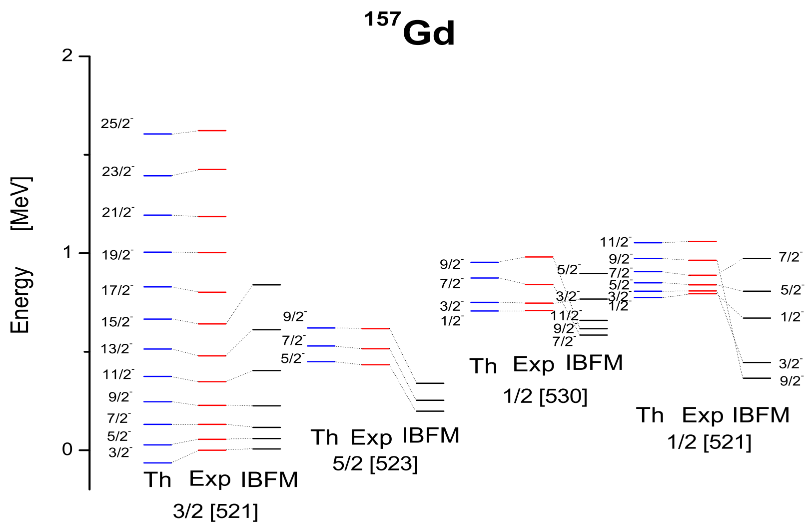

Figure 4. Comparison of the theoretical and experimental energies for the ground and first excited negative parity bands of ^{157}Gd.

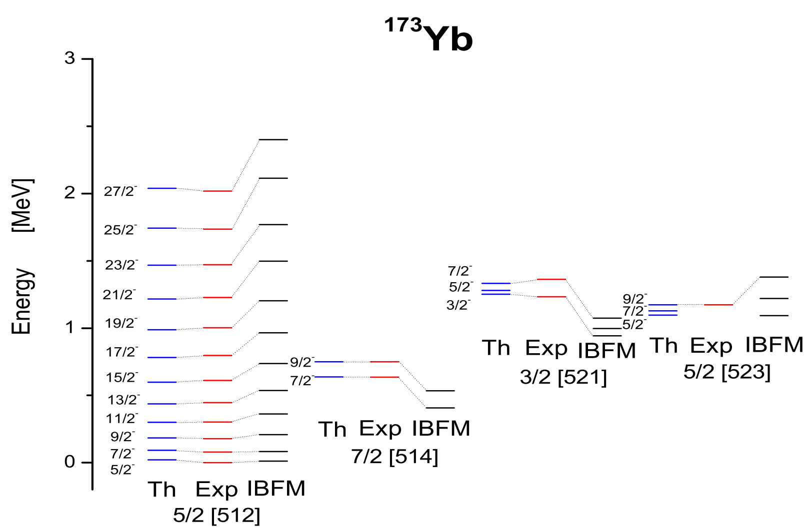

Figure 5. The same as Fig. 4, but for ^{173}Yb.

of bosons by $\Delta N = 4$ and $\Delta L = 2$. It is obvious that this term in T^{E2} (17) comes from the symplectic extension of the model. In (17) e is the effective boson charge.

The transition probabilities are by definition $SO(3)$ reduced matrix elements of transition operators T^{E2} (17) between the $|i\rangle$−initial and $|f\rangle$−final collective states (14)

$$B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \mid \langle \quad f \parallel T^{E2} \parallel i \quad \rangle \mid^2 . \qquad (19)$$

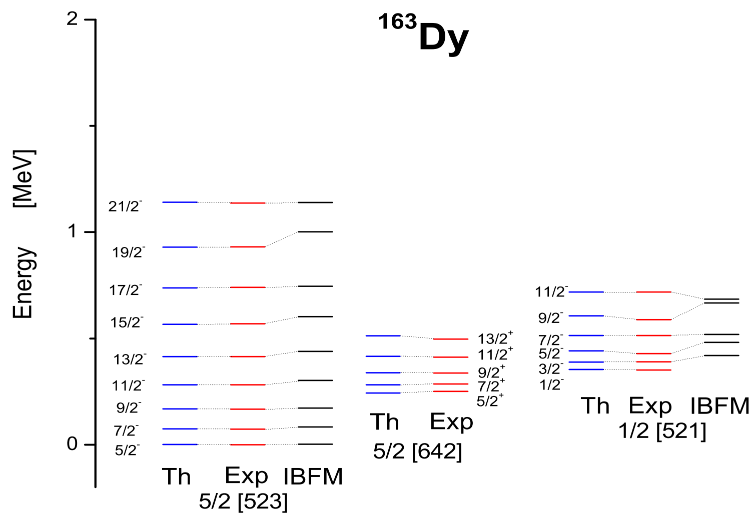

Figure 6. Comparison of the theoretical and experimental energies for the ground and first excited positive and negative parity bands of ^{163}Dy.

As was mentioned, the basis states (14) can be considered as a result of the coupling of the orbital $\mid (N,T); KLM; T_0\rangle$ (6) and spin ϕ_{jm} wave functions. Since the spin j is simply vector coupled to the orbital momentum L, the action of the transition operator T^{E2} concerns only the orbital part of the basis functions (14).

In Ref. [18] it is shown that the two main types of $B(E2)$ behavior – the enhancement or the reduction of the $B(E2)$ values within the GSB $K^\pi = 0^+$, can be reproduced simply by the change of the sign of θ. The strongly enhanced values which are an indication for increased collectivity in the high angular momentum domain are easily obtained for positive values of the parameter θ. For negative values of the parameter θ we obtain behavior similar to that of the standard $SU(3)$ one and it can be used to reproduce the well known cutoff effect. Such saturation effect is also characteristic feature of the IBM based calculations in its $SU(3)$ limit. It is shown also that although the coefficient in front of symplectic term is some orders of magnitude smaller than the $SU(3)$ contribution to the transition operator its role in reproducing the correct behavior (with or without cutoff) of the transition probabilities between the states of the GSB band is very important. For more details concerning discussed behavior of the $B(E2)$ values see [18].

In order to prove the correct predictions following from our theoretical results we apply the theory to real nuclei for which there is available experimental data for the transition probabilities [20] between the states of the ground bands up to very high angular momenta. The application actually consists of fitting the two parameters of the transition operator T^{E2} (17) to the experiment for each of the considered bands. The $B(E2)$ strengths between the negative parity states of the GSB, as were attributed to the $SU(3)$ symmetry-adapted basis states of the model, are calculated. For this $SU(3)$

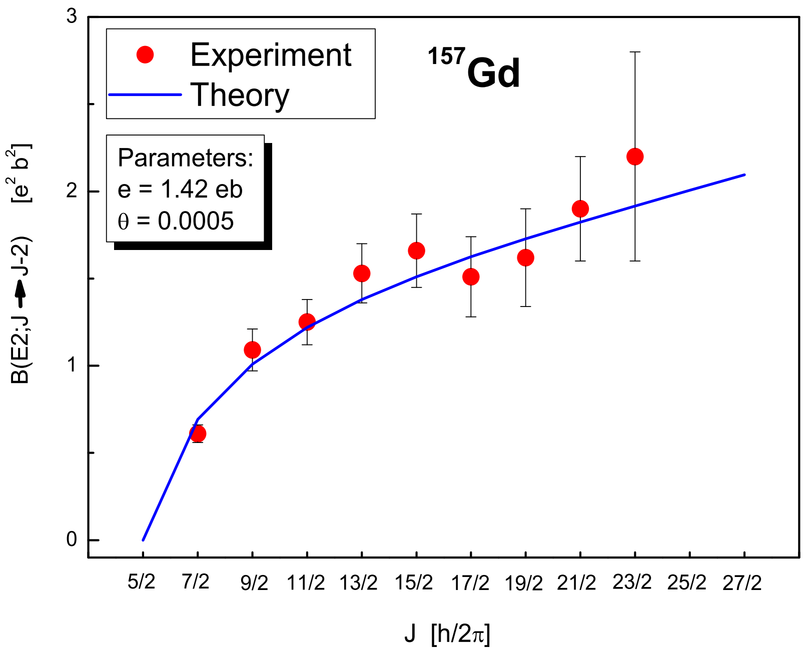

Figure 7. (Color online) Comparison of the theoretical and experimental values for the $B(E2)$ transition probabilities for the ^{157}Gd.

multiplets, the procedure for their calculations actually coincides with that given in [18]. The theoretical predictions for the nuclei ^{157}Gd and ^{163}Dy are compared with the experimental data in Figures 7 and 8. From the figures one can see that the experimental values are reproduced quite well for the both typical examples − with enhanced $B(E2)$ values (^{157}Gd) and with cutoff (^{163}Dy).

5. Conclusions

In this work we extended the dynamical symmetry group $Sp(12, R)$ of the IVBM to the orthosymplectic one $OSp(2\Omega/12, R)$. We introduced the fermion degrees of freedom by means of including a particle (quasiparticle) with spin j and exploiting the corresponding reduction $SO^F(2\Omega) \supset SU^F(2)$.

Further, the basis states of the odd systems are classified by the new dynamical symmetry (13) and the model Hamiltonian is written in terms of the first and second order invariants of the groups from the corresponding reduction chain. Hence the problem is exactly solvable within the framework of the IVBM which, in turn, yields a simple and straightforward application to real nuclear systems.

We present results that were obtained through a phenomenological fit of the models' predictions for the spectra of collective states to the experimental data for odd−A nuclei and their even-even neighbors, used as a core for the formers. The good agreement

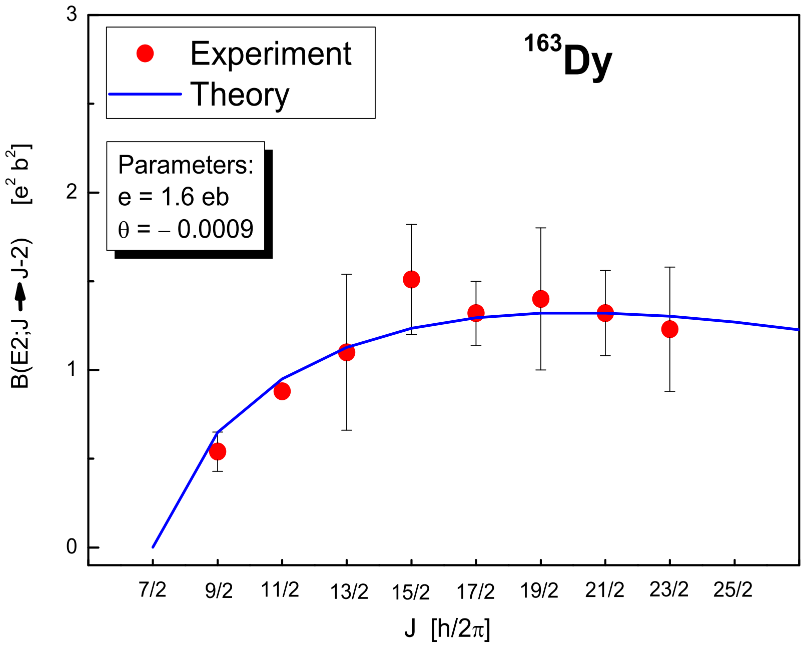

Figure 8. (Color online) The same as Fig. 7, but for ^{163}Dy.

between the theoretical and the experimental band structures confirms the applicability of the newly proposed dynamical symmetry of the IVBM. The success is based on the (ortho)symplectic structures of the model which allow the mixing of the basic collective modes −rotational and vibrational ones arising from the yrast conditions. This allows for the proper reproduction of the high spin states of the collective bands and the correct placement of the different band heads.

For two of the three isotopes considered, the $B(E2)$ transition probabilities are calculated and compared with the experimental data. The important role of the symplectic extension of the model for the correct reproduction of the $B(E2)$ behavior, observed at high angular momenta, is revealed.

The supersymmetry group $OSp(2\Omega/12, R)$ which is natural generalization of the dynamical symmetry group $Sp(12, R)$ of the IVBM could be further used to examine the correlations between the spectroscopic properties of the neighboring even-even, odd-even and odd-odd spectra of the neighboring nuclei and the underlying supersymmetry which might be considered in nuclear physics as proved experimentally [21]. These investigations are the subject of the forthcoming paper, but our preliminary results obtained in this work already suggest the typical signatures of the nuclear supersymmetry.

Acknowledgments

The author is grateful to Dr. A. I. Georgieva and Dr. V. P. Garistov for the helpful discussions. This work was supported by the Bulgarian National Foundation for scientific research under Grant Number $\Phi - 1501$.

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